Conflict Handling in Time-Dependent Subjective Networks

Thomas Wodtko, Thomas Griebel, Alexander Scheible, and Michael Buchholz

This paper has been accepted for presentation and publication at the 2024 27th International Conference on Information Fusion (FUSION), July 08 - 11, 2024, Venice, Italy. This is the accepted version of the paper, which has not been fully edited and the layout may differ from the original publication.

Citation information of the original publication:

T. Wodtko, T. Griebel, A. Scheible and M. Buchholz, ”Conflict Handling in Time-Dependent Subjective Networks,” 2024 27th International Conference on Information Fusion (FUSION), Venice, Italy, 2024, pp. 1-8, doi: 10.23919/FUSION59988.2024.10706464..

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***Abstract*— With this work, we contribute novel operators and perspectives to the field of subjective logic. We propose a novel multi-source trust revision approach enabling multi-source fusion, which considers majority tendencies to mitigate occurring conflicts. For this, the degree of conflict is extended for a multi-source use, which allows our definition of so-called conflict shares. Subsequently, combining our and existing trust revision methods, we propose a generalized trust revision approach. Extending trust revision to subjective networks describing time-dependent processes, we propose the use of sub subjective networks and further the transition to recursive subjective networks. Finally, our trust revision approach and the sub subjective network proposal are evaluated and demonstrated based on experiments, which show conflict handling favoring majorities and an efficient evaluation of time-dependent decision processes.**

I. Introduction

The process of decision-making is naturally subjective. While some aspects may seem apparent to any bystander, personal experience and trust in information sources ultimately influence every decision. More formally, information can be collected from multiple sources about the state of a random variable in a certain domain. The weight to which each source's information influences a decision may depend, besides others, on the trust in the source and the degree to which it conflicts with others.

While the process of decision-making was mathematically modeled using different notations before, the subjective logic (SL) theory recently gained importance [1]. This extension of probabilistic logic synthesizes different aspects of other approaches, e.g., the Dempster–Shafer theory of evidence (DST) [2], [3] and Bayesian networks [4]. Accordingly, it is used in various areas. For example, security applications and self-assessment algorithms have been improved using SL [5]–[7].

Our work is based on SL since it provides powerful concepts and operations our contribution builds upon. In particular, multinomial and trust opinions enable a compact mathematical representation for which fusion operators are available. The concept of trust revision (TR) [1], [8] allows the incorporation of conflicts during fusion operations. And further, subjective networks (SN) provide a graph-based structure to describe decision processes.

Parts of this work were supported by the State Ministry of Economic Affairs, Labour and Tourism Baden-Württemberg (project U-Shift II, AZ 3-433.62-DLR/60).Parts of this research have been conducted as part of the EVENTS project and other parts as part of the PoDIUM project, which both are funded by the European Union under grant agreements No. 101069614 and No. 101069547, respectively. Views and opinions expressed are, however, those of the authors only and do not necessarily reflect those of the European Union or European Commission. Neither the European Union nor the granting authority can be held responsible for them.

All authors are with the Institute of Measurement, Control and Microtechnology, Ulm University, Albert-Einstein-Allee 41, 89081 Ulm, Germany `{firstname}.{lastname}@uni-ulm.de`

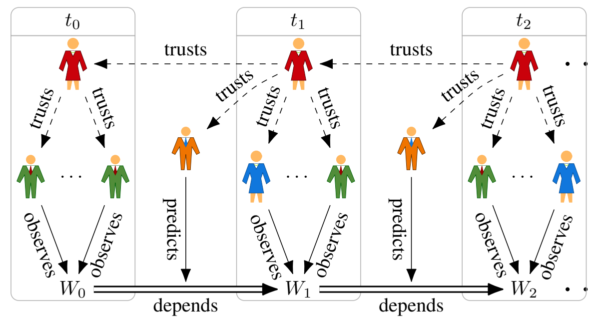


Fig. 1. The example that is used as a reference in this paper is illustrated. Alice (red) wants to decide about the weather (W) at different time steps (t_k). At every time step, a variable amount of agents (green/blue) observe the weather state and report back to Alice. Furthermore, Alice trusts each agent to a certain degree. Additionally, Alice trusts her opinion of the last time step (t_{k-1}). Generally, the weather state depends on its last state (double arrow). The way the weather changes is modeled and predicted by another agent (orange) between each time step.

Extending multi-source fusion, we propose novel multi-source TR approaches considering conflict and trust uncertainties. Consecutively, we propose a generalized TR method, combining ours and already existing TR approaches. Further, enabling the efficient description of time-dependent decision-making processes, we propose a novel perspective of SNs, which includes recursive SNs. Finally, broadening its use, we propose a way of applying TR to time-dependent decision processes, differentiating between time differences and decision conflicts. By providing these novel aspects, we contribute to the field of SL.

In this work, we consider the following example as a reference to demonstrate our new proposals: In regular time intervals, Alice decides on the current weather, which she does not directly observe. Multiple sources report their weather observation to Alice, who trusts each source to a certain degree. For demonstration purposes, the weather can either be rainy or sunny, i.e., $\mathbb{W} = \{\text{rainy}, \text{sunny}\}$. The number of available sources at each time step t_k may vary, and for every step, a prediction of weather changes is available. The scenario is illustrated in Fig. 1.

To the best of our knowledge, available tools in SL do not allow an efficient description of the described example. SNs would indefinitely grow over time, and only one TR approach is available. As shown later, the TR approach lacks flexibility and cannot be adjusted to a specific task.

For better readability and self-containment of this paper, we provide detailed definitions of SL components in the foundations in Section II. Note that these components are the basis of our newly developed methods. Respectively, the context of *Definitions* is already available in the literature. In contrast, our contributions are presented in the form of *Proposals* and *Theorems* in Section III & IV.

Summarizing our work in this paper, we propose

- novel multi-source TR approaches,
- a new perspective on time-dependent SNs, and
- a TR in time dependent decision processes.

II. Subjective Logic

This section outlines the fundamentals of SL used in this work. For a more detailed explanation and overview, the reader shall be referred to [1]. SL is a mathematical framework that explicitly represents statistical uncertainty [1], comparable to the DST [2], [3].

A. Representation and Similarity

The key aspect of SL is the representation of opinions. Typically, a multinomial opinion embodies information about a discrete random variable X for each event x in the sample space $\mathbb{X}$, in terms of belief, uncertainty, and base rate.

Definition 1 (Multinomial Opinion [1]). *Consider a random variable X in the finite domain $\mathbb{X}$ with cardinality $\mathcal{X} = |\mathbb{X}| \geqslant 2$. A multinomial opinion can be defined as an ordered triple $\omega_X = (\boldsymbol{b}_X, u_X, \boldsymbol{a}_X)$ with*

$$\boldsymbol{b}_X(x) : \mathbb{X} \mapsto [0,1], \qquad 1 = u_X + \sum_{x \in \mathbb{X}} \boldsymbol{b}_X(x)\,, \tag{1a}$$

$$\boldsymbol{a}_X(x) : \mathbb{X} \mapsto [0,1], \qquad 1 = \sum_{x \in \mathbb{X}} \boldsymbol{a}_X(x)\,. \tag{1b}$$

The belief mass distribution $\boldsymbol{b}_X$ over $\mathbb{X}$ reflects the belief in each event, the uncertainty mass $u_X \in [0,1]$ signifies the lack of evidence, and the base rate distribution $\boldsymbol{a}_X$ over $\mathbb{X}$ represents the prior probability for each event.

The projected probability $\boldsymbol{P}_X(x) : \mathbb{X} \mapsto [0,1]$ is used to project a multinomial opinion into a classical probability distribution. It is defined by

$$\boldsymbol{P}_X(x) = \boldsymbol{b}_X(x) + \boldsymbol{a}_X(x) u_X\,, \tag{2}$$

and represents the expected outcome of the opinion in a classical probability space. For illustrations of opinions, the reader is referred to [1]. To visualize binomial opinions, the barycentric triangle, which is displayed in Fig. 2, is used.

Given two opinions on the same variable X, the *degree of conflict* (DC) is used to calculate their difference.

Definition 2 (Degree of Conflict [1]). *Let ω_X^A and ω_X^B be multinomial opinions of the same variable $X \in \mathbb{X}$ provided by source A and B, respectively. The degree of conflict between these two opinions is defined as* $\mathrm{DC}\left(\omega_X^A, \omega_X^B\right) \in [0,1]$ *with*

$$\mathrm{DC}\left(\omega_X^A, \omega_X^B\right) = \mathrm{PD}\left(\omega_X^A, \omega_X^B\right) \cdot \mathrm{CC}\left(\omega_X^A, \omega_X^B\right)\,, \tag{3}$$

using the projected distance $\mathrm{PD}\left(\omega_X^A, \omega_X^B\right) = \frac{1}{2}\sum_{x \in \mathbb{X}} |\boldsymbol{P}_X^A(x) - \boldsymbol{P}_X^B(x)| \in [0,1]$ *and the conjunctive certainty* $\mathrm{CC}\left(\omega_X^A, \omega_X^B\right) = \left(1 - u_X^A\right)\left(1 - u_X^B\right) \in [0,1]$.

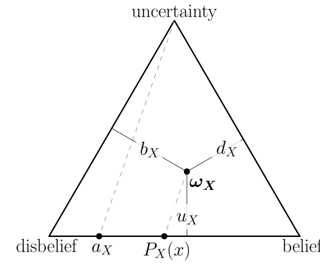


Fig. 2. A binomial opinion ω_X is illustrated in a barycentric triangle. The three axes of *belief*, *disbelief*, and *uncertainty* are represented by b_X, d_X, and u_X, respectively, and a_X is the prior projecting ω_X to $P_X(x)$.

The magnitude of the DC is reciprocal to the similarity of the two opinions ω_X^A and ω_X^B. In this work, the DC is extended to calculate the conflict of multiple sources about a common variable $X \in \mathbb{X}$, and the importance of this in multi-source TF is further elaborated in Section II-D & III.

B. Multi-Source Information Fusion

Information from multiple sources about a common random variable must be combined when describing a fusion process. For the information fusion of several opinions, various operators are available in literature [1], [8], [9]. The idea is to bring together a set of opinions to reach a joint conclusion about a specific task through the use of a fusion operator.

Definition 3 (Multi-Source Fusion [8], [9]). *Let $\mathbb{S}$ be a set of $N \in \mathbb{N}$ sources represented by $S_1, \ldots, S_N$. Further, let $W_X^{\mathbb{S}} = \{\omega_X^{S_1}, \ldots, \omega_X^{S_N}\}$ be a set of opinions which contains an opinion of each source about a common variable X. Multi-source fusion describes the process of reaching a joint conclusion given the set of opinions $W_X^{\mathbb{S}}$.*

Choosing a suitable fusion operator is vital and depends on the task at hand. Therefore, the author of [1] proposes certain criteria to be considered for this. Since used later, the multi-source fusion operators *cumulative belief fusion* (CBF) and *averaging belief fusion* (ABF) are defined next. For implementation details, the reader is referred to [8], [9].

Definition 4 (Cumulative Belief Fusion [8]). *The CBF assumes that incorporating additional, independent sources of evidence will accumulate and strengthen the overall belief. Given $\mathbb{S}$ and $W_X^{\mathbb{S}}$ from Definition 3, the CBF of all sources in $\mathbb{S}$ is referred to $\diamond(\mathbb{S})$ such that $\omega_X^{\diamond(\mathbb{S})} = \left(\boldsymbol{b}_X^{\diamond(\mathbb{S})}, u_X^{\diamond(\mathbb{S})}, \boldsymbol{a}_X^{\diamond(\mathbb{S})}\right)$. The general notation of CBF is abbreviated as*

$$\omega_X^{\diamond(\mathbb{S})} = \oplus(W_X^{\mathbb{S}}) = \bigoplus_{S \in \mathbb{S}} \left(\omega_X^S\right) = \omega_X^{S_1} \oplus \ldots \oplus \omega_X^{S_N}\,. \tag{4}$$

Here, associativity, commutativity, and non-idempotent can be verified [8].

Definition 5 (Averaging Belief Fusion [8]). *The ABF takes into account the interdependence between sources and assumes that adding more sources does not necessarily lead to a stronger conclusion with lower uncertainty. Given $\mathbb{S}$ and $W_X^{\mathbb{S}}$ from Definition 3, the ABF of all sources in $\mathbb{S}$ is referred*

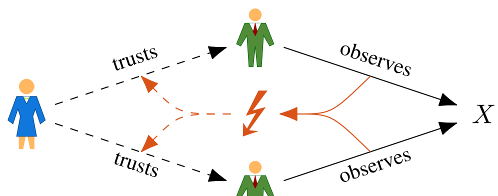


Fig. 3. Inspired by [1], an exemplary TF scenario is depicted for which TR is applied. Two agents (green) observe the state of X and report back to a third agent (blue), which trusts each source to a certain degree. TR describes the way of updating the trust (dashed arrows) based on the conflict (lightning) of the two source opinions (normal arrow).

to $\underline{\diamond}(\mathbb{S})$ such that $\omega_X^{\underline{\diamond}(\mathbb{S})} = \left(\boldsymbol{b}_X^{\underline{\diamond}(\mathbb{S})}, u_X^{\underline{\diamond}(\mathbb{S})}, \boldsymbol{a}_X^{\underline{\diamond}(\mathbb{S})}\right)$. The general notation of ABF is abbreviated as

$$\omega^{\underline{\diamond}(\mathbb{S})} = \underline{\oplus}(W_X^{\mathbb{S}}) = \underline{\bigoplus_{S \in \mathbb{S}}} \left(\omega_X^S\right) . \tag{5}$$

Here, commutativity and idempotent can be verified [8], [9]. It shall be noted that the consecutive execution of ABF of two opinions is non-associative.

C. Notion of Trust

Besides psychological interpretations, trust in a source may represent, for example, some security aspects [5] or the integrity of sensor data. In [1], Jøsang discusses different perspectives of trust. The computational trust in SL represents reliability trust, which is defined as

Definition 6 (Reliability Trust [1]). *Reliability trust is the subjective belief with which an entity, A, expects that another entity, B, performs a given action on which A's welfare depends [1].*

In the following, trust always refers to the computational trust of SL. Given an agent A that trusts a source B which observes a variable $X \in \mathbb{X}$, A yields an opinion over X based on trust discounting. For this, the trust of A in B is represented by a binomial opinion ω_B^A, called trust opinion. This is combined with the opinion ω_X^B of B, called evidential belief or source opinion, by applying the trust discount operation.

Definition 7 (Trust Discount [1]). *Let ω_B^A be the trust of the agent A in the source B. Further, let ω_X^B be the opinion of B over the variable $X \in \mathbb{X}$. The opinion of A over X denoted by $\omega_X^{[A;B]} = \omega_B^A \otimes \omega_X^B$ is defined by*

$$\omega_X^{[A;B]} : \begin{cases} \boldsymbol{b}_X^{[A;B]}(x) = \boldsymbol{P}_B^A \boldsymbol{b}_X^B(x) \, , \\ u_X^{[A;B]} = 1 - \boldsymbol{P}_B^A \sum\limits_{x \in \mathbb{X}} \boldsymbol{b}_X^B(x) \, , \\ \boldsymbol{a}_X^{[A;B]}(x) = \boldsymbol{a}_X^B(x) \, . \end{cases} \tag{6}$$

D. Trust Revision

To fuse multiple opinions, several operators have been introduced in Section II-B. However, additional aspects must be considered when trust is involved in the fusion process. Before applying one of the fusion operators described earlier, all opinions are discounted based on the trust in its source. Then, the fusion operation is applied to the set of discounted opinions as before.

Definition 8 (Trust Fusion [1]). *Given $\mathbb{S}$ and $W_X^{\mathbb{S}}$ from Definition 3, let A be an entity, ω_S^A the trust of A in a source $S \in \mathbb{S}$, and $W_{\mathbb{S}}^A = \{\omega_S^A \,|\, S \in \mathbb{S}\}$ a set containing the trust of A in each source. The trust fusion (TF) with respect to a fusion operation, e.g., the CBF, is then defined by*

$$\omega_X^{\diamond([A;\mathbb{S}])} = \bigoplus_{S \in \mathbb{S}} \left(\omega_S^A \otimes \omega_X^S\right) . \tag{7}$$

As shown in Fig. 3, in a TF scenario, the TR describes the trust adjustment based on the source opinions' conflict. The revision is usually based on revision factors (RF), sometimes also called revision weight.

Definition 9 (Trust Revision [1], [8]). *Given $\mathbb{S}$, A, and $W_{\mathbb{S}}^A$ from Definition 8, and revision factors for each trust opinion, i.e., $\mathrm{RF} : W_{\mathbb{S}}^A \mapsto [0, 1]$, the TR yields updated trust opinions $\breve{\omega}_S^A$, $\forall S \in \mathbb{S}$ defined by*

$$\breve{\omega}_S^A : \begin{cases} \breve{b}_S^A = b_S^A - b_S^A \cdot \mathrm{RF}(\omega_S^A) \, , \\ \breve{d}_S^A = d_S^A + (1 - d_S^A) \cdot \mathrm{RF}(\omega_S^A) \, , \\ \breve{u}_S^A = u_S^A - u_S^A \cdot \mathrm{RF}(\omega_S^A) \, , \\ \breve{a}_S^A = a_S^A \, , \end{cases} \tag{8}$$

where b denotes the belief and d the disbelief, which represent the belief masses of a binomial opinion.

One TR approach is described in [1] for two opinions. Another approach is proposed in [8] for two or more opinions. Since the multi-source approach is used as a comparison it is outlined here.

Definition 10 (Fusion Reference based Trust Revision [8]). *Given $\mathbb{S}$, $W_{\mathbb{S}}^A$, and $W_X^{\mathbb{S}}$ from Definition 8, a fusion reference (FR) ω_X^{FR} is calculated by*

$$\omega_X^{FR} = \omega_X^{\underline{\diamond}([A;\mathbb{S}])} . \tag{9}$$

The belief conflict $\mathrm{BC} : W_X^{\mathbb{S}} \mapsto [0, 1]$ for each source opinion $\omega_X^S \in W_X^{\mathbb{S}}$ is defined in relation to the FR as

$$\mathrm{BC}\left(\omega_X^S\right) = \mathrm{DC}\left(\omega_X^S, \omega_X^{FR}\right) . \tag{10}$$

Futher, the maximum conflict (MC) and average conflict (AC) are calculated by

$$\mathrm{MC}\left(W_X^{\mathbb{S}}\right) = \max_{S \in \mathbb{S}} \mathrm{BC}\left(\omega_X^S\right) , \tag{11}$$

$$\mathrm{AC}\left(W_X^{\mathbb{S}}\right) = \frac{1}{|W_X^{\mathbb{S}}|} \sum_{S \in \mathbb{S}} \mathrm{BC}\left(\omega_X^S\right) . \tag{12}$$

Then, the RFs of the TR are defined as

$$\mathrm{RF}(\omega_S^A) = \begin{cases} \dfrac{\mathrm{MC}\left(W_X^{\mathbb{S}}\right) \cdot d}{\mathrm{MC}\left(W_X^{\mathbb{S}}\right) - \mathrm{AC}\left(W_X^{\mathbb{S}}\right)} & \text{if } d > 0 \, , \\ 0 & \text{else} \, , \end{cases} \tag{13}$$

with $d = \mathrm{BC}\left(\omega_X^S\right) - \mathrm{AC}\left(W_X^{\mathbb{S}}\right)$.

Using the TR approach from Definition 10 only alters the trust in sources whose BC is greater than AC. Thus, the fusion operator choice for the reference fusion is of importance. Depending on the operator, trust may change or stay untouched based on the same sets of opinions and trust.

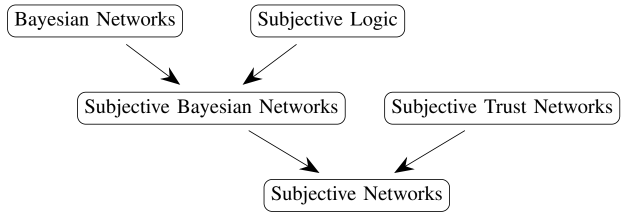


Fig. 4. The general idea of SNs is illustrated.

E. Conditional Reasoning

SL also provides tools to model conditional reasoning, namely deduction and abduction. For this work, only the deduction is of importance and, thus, introduced in the following. The SL deduction describes the reasoning over a variable based on observations of dependent variables and extends the probabilistic deduction. For better comprehensibility, the binomial deduction is introduced first, followed by the multinomial deduction.

Definition 11 (Binomial Deduction [1]). *Let $X \in \mathbb{X}$ and $Y \in \mathbb{Y}$ be variables in binomial domains, i.e., $\mathcal{X} = \mathcal{Y} = 2$. Further, let ω_X be an evidential belief in X and the opinions $\omega_{Y|x}$ and $\omega_{Y|\overline{x}}$ be the conditional opinions of Y given that X is true or false, respectively. Then, the binomial deduction is defined by*

$$\omega_{Y||X} = \omega_X \circledcirc \left(\omega_{Y|x}\, , \omega_{Y|\overline{x}}\right) \, , \tag{14}$$

where $\omega_{Y||X}$ denotes the deduced binomial opinion on Y.

In the binomial case, the variable X may only be one of two states; thus, two conditional opinions are required to express the conditional reasoning on Y. In contrast, for each dimension of X, a conditional opinion on Y must be available to allow multinomial deduction.

Definition 12 (Multinomial Deduction [1]). *Let $X \in \mathbb{X}$ and $Y \in \mathbb{Y}$ be multinomial variables in different multinomial domains with cardinality $\mathcal{X}$ and $\mathcal{Y}$, respectively. Further, let $\boldsymbol{\omega}_{Y|X} = \{\omega_{Y|x_i} \,|\, i = 1, \dots, \mathcal{X}\}$ be a set of conditional opinions with a conditional opinion on Y for each possible event in $\mathbb{X}$. Then, the multinomial deduction is defined by*

$$\omega_{Y||X} = \omega_X \circledcirc \boldsymbol{\omega}_{Y|X} \, , \tag{15}$$

where $\omega_{Y||X}$ denotes the deduced multinomial opinion on Y.

Both binomial and multinomial deduction are graphically explained in [1] to which the reader shall be referred to.

F. Subjective Networks

In [1], SNs are introduced as a graph-based structure with agents or sources and variables combined with conditional and trust opinions. The general idea is illustrated in Fig. 4.

Definition 13 (Subjective Network [1]). *An SN describes a decision process and is a directed acyclic graph (DAG) that contains agents and variables. The connections between agents represent trust, between agents and variables observations, and between variables conditional connections.*

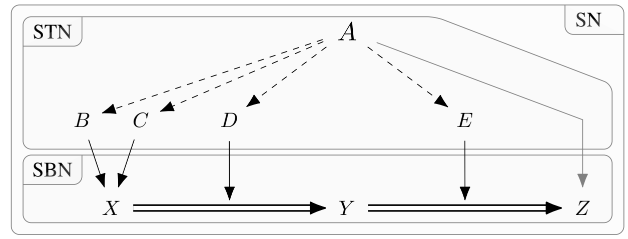


Fig. 5. An SN example similar to [1] is depicted. Dashed arrows represent trust relations, normal arrows represent belief relations, and double arrows represent conditional relations. The goal for A is to decide about Z given the opinions of B, C, D, and E together with a respective trust opinion for each of them. Here, the combination of STNs and SBNs to SNs is visible. The upper part of the graph, which includes the trust relations of A to other agents, is an STN, and the lower part of the graph, which includes the agent's observations and conditional reasoning, is an SBN.

The limitation of Bayesian networks (BN), which require precise probabilities as inputs, is mitigated using SL opinions, and conditional probability distributions are replaced by conditional subjective opinions, leading to the concept of subjective Bayesian networks (SBN). Then, in combination with subjective trust networks (STN), agents can express different conditional and evidential opinions on the same conditionals and variables in the network, leading to SN. An SN example similar to [1] is shown in Fig. 5. It illustrates the decision process for A on Z. Each relation in Fig. 5, namely, trust, belief, and conditional relations, can be denoted with its respective SL opinion. Given the structure of Fig. 5, the decision process can now be expressed using the previously defined SL operators. First, the opinion from A on X is calculated by

$$\omega_X^{\diamond([A;\{B,C\}])} = \left(\omega_B^A \otimes \omega_X^B\right) \oplus \left(\omega_C^A \otimes \omega_C^B\right) \, . \tag{16}$$

Here, CBF is exemplarily used for fusion; however, any fusion operation can be used depending on the task. Next, the two-step deduction, from X via Y to Z, is calculated with

$$\omega_{Z||X}^A = \left(\omega_X^{\diamond([A;\{B,C\}])} \circledcirc \boldsymbol{\omega}_{Y|X}^{[A;D]}\right) \circledcirc \boldsymbol{\omega}_{Z|Y}^{[A;E]} \, , \tag{17}$$

where $\boldsymbol{\omega}_{Y|X}^{[A;D]} = \left(\omega_D^A \otimes \boldsymbol{\omega}_{Y|X}^D\right)$ and $\boldsymbol{\omega}_{Y|X}^{[A;E]} = \left(\omega_E^A \otimes \boldsymbol{\omega}_{Z|Y}^E\right)$ represent the discounted conditional opinions.

III. Multi-Source Conflict Handling

The conflict of two opinions can be evaluated using the DC (see Definition 2) in fusion and TF scenarios. Handling this conflict means taking action in case of high conflict magnitudes. Jøsang presents a TR approach in [1] for two source opinions, where the trust in sources is altered based on uncertainty differentials. By using these differentials, the conflict is assigned to each source based on its trust uncertainty in relation to the others. Thus, trust uncertainties are the main values that influence this TR behavior. This work will extend the uncertainty differential-based TR approach for more than two sources. Since the extended approach is still only based on trust uncertainties, no relations between source opinions can be considered. In contrast, using a FR, Jøsang et al. present a multi-source TR approach in [8]. The trust is revised based on the conflict between a source opinion and the FR. While this allows a definition for more than two source opinions, choosing a suitable operator is vital to the TR and must be done task-specifically. This work aims to find a TR approach that incorporates majority tendencies within the source opinions without requiring an FR. With this, the approach is independent of prior fusion operator choices. Although at first glance, it seems that the multi-source TR approach presented by Jøsang et al. [8] already incorporates majority tendencies to a certain degree, our experiments in Section V will demonstrate different behaviors and the advantage of our approach based on the introductory example.

A. Degree of Conflict

In this section, we propose an extension of the DC definition by Jøsang in [1] to allow more than two opinions to be compared. The goal is to omit the use of any FR to provide an approach that is not task-specific. Instead, the conflict of each possible opinion pair is considered.

Proposal 1 (Multi-Source Degree of Conflict). *Let $\mathbb{S}$ be a set of $N \in \mathbb{N}$ sources observing a common random variable $X \in \mathbb{X}$ and $W_X^{\mathbb{S}} = \{\omega_X^S \mid S \in \mathbb{S}\}$. Further, let A be an agent and $W_{\mathbb{S}}^A$ a set that contains the trust of A in each source. Then, the accumulated degree of conflict (DC_{acc}) is defined as*

$$\mathrm{DC}_{\mathrm{acc}}\left(W_{\mathbb{S}}^A, W_X^{\mathbb{S}}\right) = \sum_{\substack{\{S_i,S_j\}\in\mathbb{S}\times\mathbb{S}\\ i<j}} \mathrm{DC}\left(\omega_X^{[A;S_i]}, \omega_X^{[A;S_j]}\right) . \tag{18}$$

The average degree of conflict ($\mathrm{DC}_{\mathrm{avg}}$) normalizes the $\mathrm{DC}_{\mathrm{acc}}$ with the total number of opinion pairs and is defined by

$$\mathrm{DC}_{\mathrm{avg}}\left(W_{\mathbb{S}}^A, W_X^{\mathbb{S}}\right) = \frac{1}{c}\,\mathrm{DC}_{\mathrm{acc}}\left(W_{\mathbb{S}}^A, W_X^{\mathbb{S}}\right) , \tag{19}$$

where $c \in \mathbb{N}$ is total number of pairs with $c = \frac{N(N-1)}{2}$.

In the special case of $N = 2$, the $\mathrm{DC}_{\mathrm{acc}}$ and the $\mathrm{DC}_{\mathrm{avg}}$ equal the DC. Next, conflict shares (CS) are proposed to determine majority tendencies within the sources. An CS assigns each source a share to which it is responsible for the current conflict. For this, the $\mathrm{DC}_{\mathrm{avg}}$ is considered with respect to a $\mathrm{DC}_{\mathrm{avg}}$ calculation without a certain source's opinion. A change in the conflict indicates that the source's opinion is either inhibiting or favoring the conflict.

Proposal 2 (Conflict Share). *Given $\mathbb{S}$, $W_X^{\mathbb{S}}$, A, and $W_{\mathbb{S}}^A$ from Proposal 1, the* CS : $W_{\mathbb{S}}^A \mapsto [0,1]$ *of a trust opinion $\omega_S^A \in W_{\mathbb{S}}^A$ is defined as*

$$\mathrm{CS}(\omega_S^A) = \max\left(0\,,\,1 - \frac{\mathrm{DC}_{avg}\left(W_{\mathbb{S}\setminus S}^A, W_X^{\mathbb{S}\setminus S}\right)}{\mathrm{DC}_{avg}\left(W_{\mathbb{S}}^A, W_X^{\mathbb{S}}\right)}\right) . \tag{20}$$

Using the DC_{avg} over the DC_{acc} automatically considers the varying number of pairs within the calculation.

The $\mathrm{DC}_{\mathrm{avg}}$ value can either increase, decrease, or stay the same when a source is not considered compared to the $\mathrm{DC}_{\mathrm{avg}}$ of all sources. If the conflict increases or does not change, a source is assumed not to be responsible for the overall conflict potential since it has a rather mitigating effect. Hence, the CS is clipped to the interval $[0, 1]$.

B. Trust Revision

In this section, we first extend the TR concept based on uncertainty differentials (UD) of [1] to the multi-source case. Then, we use CSs to propose a novel TR approach. In [1], UD only considers the uncertainty of trust into two sources. More specifically, the UD for a source yields the proportion of its uncertainty relative to the sum of uncertainties of both. This can be extended to allow more than two sources.

Proposal 3 (Multi-Source Uncertainty Differentials). *With $\mathbb{S}$, A, and $W_{\mathbb{S}}^A$ from Proposal 1, the multi-source* UD : $W_{\mathbb{S}}^A \mapsto [0,1]$ *is defined by*

$$\mathrm{UD}\left(\omega_S^A\right) = \frac{u_S^A}{\sum\limits_{S^\star\in\mathbb{S}} u_{S^\star}^A} \tag{21}$$

Using the proposed UD extension from Proposal 1 and following [1], RFs can now be calculated using either $\mathrm{DC}_{\mathrm{acc}}$ or $\mathrm{DC}_{\mathrm{avg}}$.

Proposal 4 (Multi-Source Trust Revision with Uncertainty Differentials). *Given $\mathbb{S}$, $W_X^{\mathbb{S}}$, A, and $W_{\mathbb{S}}^A$ from Proposal 1, the extended* RF*s, i.e.,* $\mathrm{RF}_{acc}^{\mathrm{UD}} : W_{\mathbb{S}}^A \mapsto [0,1]$ *and* $\mathrm{RF}_{avg}^{\mathrm{UD}} : W_{\mathbb{S}}^A \mapsto [0,1]$*, are defined by*

$$\mathrm{RF}_{acc}^{\mathrm{UD}}\left(\omega_S^A\right) = \mathrm{UD}\left(\omega_S^A\right) \cdot \mathrm{DC}_{acc}\left(W_{\mathbb{S}}^A\,,\,W_X^{\mathbb{S}}\right) , \tag{22}$$

$$\mathrm{RF}_{avg}^{\mathrm{UD}}\left(\omega_S^A\right) = \mathrm{UD}\left(\omega_S^A\right) \cdot \mathrm{DC}_{avg}\left(W_{\mathbb{S}}^A\,,\,W_X^{\mathbb{S}}\right) . \tag{23}$$

Next, using our proposed CS method from Proposal 2 leads to a novel TR approach favoring majority tendencies within a set of sources while still considering each source's trust, making it even more powerful.

Proposal 5 (Multi-Source Trust Revision with Conflict Shares). *With $\mathbb{S}$, $W_X^{\mathbb{S}}$, A, and $W_{\mathbb{S}}^A$ from Proposal 1, the extended* RF*s, i.e.,* $\mathrm{RF}_{acc}^{\mathrm{CS}} : W_{\mathbb{S}}^A \mapsto [0,1]$ *and* $\mathrm{RF}_{avg}^{\mathrm{CS}} : W_{\mathbb{S}}^A \mapsto [0,1]$ *using the* CS *concept are defined by*

$$\mathrm{RF}_{acc}^{\mathrm{CS}}\left(\omega_S^A\right) = \mathrm{CS}\left(\omega_S^A\right) \cdot \mathrm{DC}_{acc}\left(W_{\mathbb{S}}^A\,,\,W_X^{\mathbb{S}}\right) , \tag{24}$$

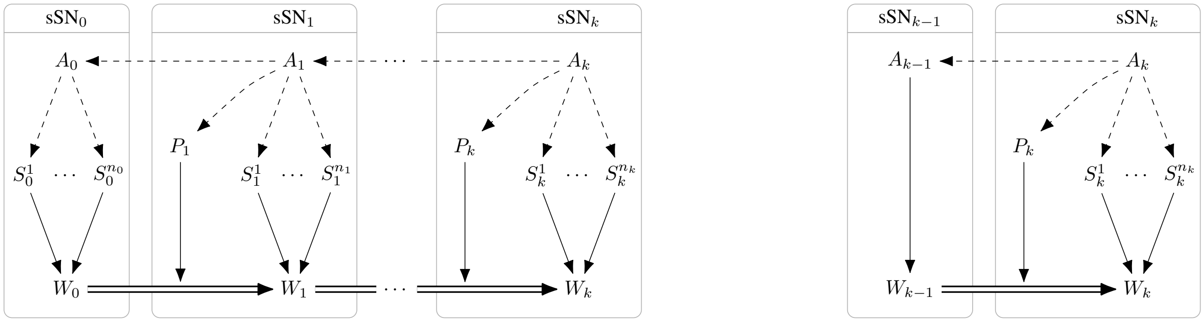
$$\mathrm{RF}_{avg}^{\mathrm{CS}}\left(\omega_S^A\right) = \mathrm{CS}\left(\omega_S^A\right) \cdot \mathrm{DC}_{avg}\left(W_{\mathbb{S}}^A\,,\,W_X^{\mathbb{S}}\right) . \tag{25}$$

Given the different TR approaches, it is desirable and advantageous to combine them into a generalized TR (gTR) concept.

Proposal 6 (Generalized Multi-Source Trust Revision). *Given $M \in \mathbb{N}$ different multi-source TR approaches, weights for each approach $\boldsymbol{\alpha} \in \mathbb{R}_{\geqslant 0}^M$, and $\mathbb{S}$, $W_X^{\mathbb{S}}$, A, and $W_{\mathbb{S}}^A$ from Proposal 1, the gTR is defined by*

$$\mathrm{RF}_g\left(\omega_S^A\right) = \min\left(1\,,\,\sum_{i=0}^{M} \alpha_i\,\mathrm{RF}_i\left(\omega_S^A\right)\right) . \tag{26}$$

Limiting the sum of weights seems reasonable, i.e., $\sum_i \alpha_i \leqslant 1$.



(a) Growing SN of the weather forecast example.

(b) Recursive SN of the weather forecast example.

Fig. 6. The example described in Section I is illustrated as SN in two variants. In 6a, the SN grows over time, while the recursive definition in 6b shifts its reference time point and makes it computationally more efficient.

IV. Time-Dependent Subjective Networks

In this section, we propose a new perspective on SNs that allows us to use SNs to efficiently model processes like our motivation weather forecast example illustrated in Fig. 1.

Generally, an SN is a DAG [1]. Thus, by definition, each agent within the graph is uniquely represented by a node. This means that edges in relation to an agent are connected to the same node representing this agent. When modeling a time-independent decision process, all considered information is available, and agents do not change their observations and corresponding opinions during the process. It, thus, makes sense to connect every edge to the same node from a decision making point of view. In contrast, however, when dealing with time-dependent decision processes, agents may appear multiple times, and the graph structure is increasing over time. Respectively, new nodes and edges must be added, and to accurately describe the decision process, adding edges to already considered nodes alters results at places where it is not supposed to. Hence, we propose to consider each time step within a time-dependent as its own sub-SN (sSN).

Proposal 7 (Time-Dependent Subjective Networks). *To describe a decision process over time using an SN, we propose to consider time-specific parts of the SN as their own sub-SN (sSN), which at time step k is denoted by sSN$_k$. For this, the following aspects must be considered:*

- *Agents may appear in multiple sSN if and only if their connections are independent of past and future appearances; thus, they are de facto considered as different agents at different time steps.*
- *The value of random variables must not change over time and, thus, must not appear multiple times.*
- *Connections between nodes of different sSN must follow the same rules as within each SN with the addition that all relations must maintain the causality, i.e., no connection to future sSN is allowed when evaluating the decision process at a certain time step.*

Using the notation presented in Section II-F and our proposed sSN division, the introductory example of Fig 1 is presented in Fig. 6a. Still, the structure of the SN grows over time as more and more nodes and variables are added to the graph. While this accurately describes the decision process, it is complex to maintain this structure for long time periods.

Theorem 1 (Markov Property of Time-Dependent Subjective Networks). *The Markov property holds if any sSN within an SN has connections only to its preceding sSN.*

Proof. By definition, an sSN represents a part of an SN at a certain time step. Thus, given that an sSN has connections only to its preceding sSN, every entity within an sSN may only depend on the current and the previous time step. ☐

Theorem 2 (Recursive Time-Dependent Subjective Networks). *If the Markov property holds for a time-dependent SN, and the structure within a sSN does not change over time, the SN can be represented in a recursive manner.*

Proof. Given a time step k, if the Markov property holds, the sSN$_{k+1}$ may only depend on values of sSN$_k$. However, due to causality, all values sSN$_{k+1}$ depends on can be evaluated at the time step k. Thus, if the structure of each sSN is the same, the sSN at any time step can fully be evaluated based on previously evaluated sSN. ☐

Following Theorem 2, the SN of Fig. 6a is illustrated in a recursive manner in Fig. 6b. Formulary, using CBF and TR, the opinion of A_k on W_k is defined as

$$\begin{aligned}\omega_{W_k}^{A_k} = \Big(\Big(\omega_{A_{k-1}}^{A_k} \otimes \omega_{W_{k-1}}^{A_{k-1}}\Big) \circledcirc \Big(\breve{\omega}_{P_k}^{A_k} \otimes \boldsymbol{\omega}_{W_k|W_{k-1}}^{P_k}\Big)\Big) \\ \oplus\, \breve{\omega}_{W_k}^{[A_k,S_k^1]} \oplus \cdots \oplus \breve{\omega}_{W_k}^{[A_k,S_k^{n_k}]}\,.\end{aligned} \tag{27}$$

Here, the advantage of a recursive definition becomes apparent since it can be defined compact, which allows an efficient online evaluation.

Remark 1. *Using TR in the trusted fusion described in Eq. (27) has not been clearly specified yet. We propose to assign any conflict-based RF to the trust of the conditional opinion as it is mainly responsible for possible conflicts during fusion. Altering the trust in previous opinions based on, e.g., the time difference seems more reasonable.*

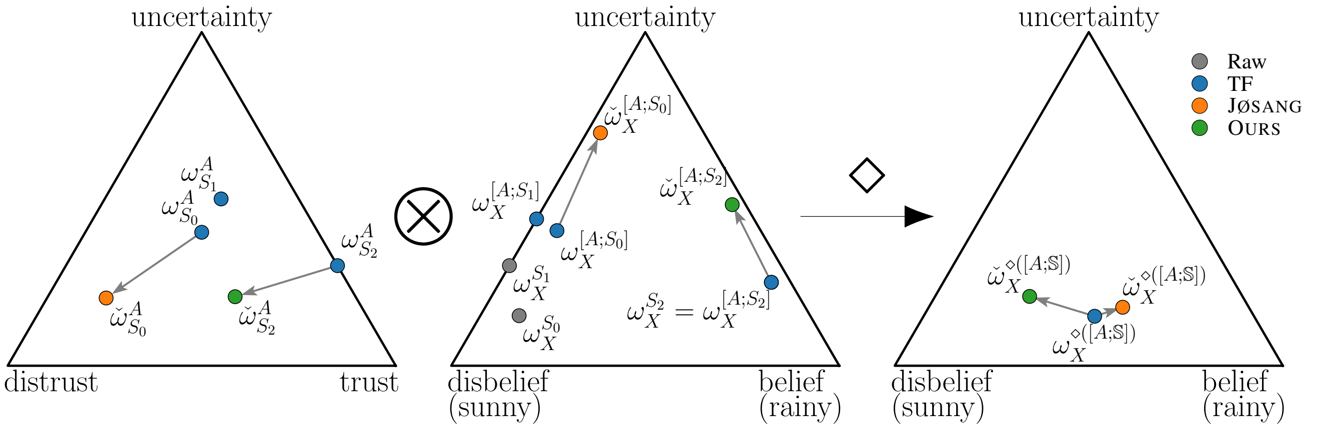


Fig. 7. TR results for a single step of the weather forecast experiment. The left triangle shows the trust of A into the sources S_i, the middle triangle the source opinions, and the right triangle the fusion results. Gray-colored opinions are original source opinions, blue-colored opinions represent a TF without TR, and green- and orange-colored opinions illustrate the changes when using TR fusion with OURS and JØSANG, respectively. The base rate $a_X(x) = 1$ is used for all trusts, the base rate $a_X(x) = 0.5$ for all source opinions, and, for visualization purposes, all RF have been scaled by a factor of two.

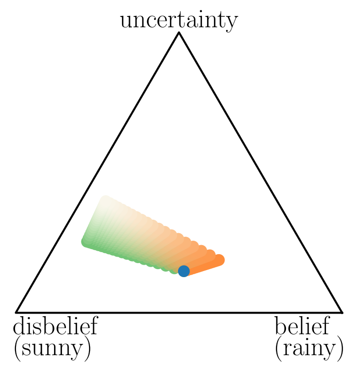
V. EXPERIMENTS

In this section, we investigate the impacts of our proposals based on the weather forecast scenario described in Section I. First, our proposed TR approach is evaluated in comparison with the existing approach of Jøsang [8]. Then, we show and discuss the results of our time-dependent experiment.

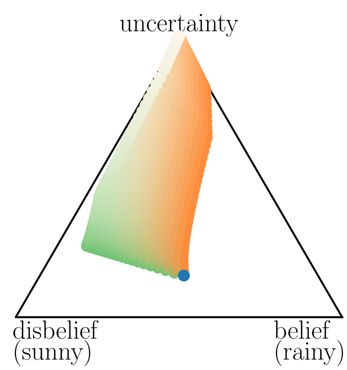
A. Trust Revision

A time-independent experiment allows us to show more detailed insights into our TR approach. For this, the time-independent weather forecast scenario with specially selected values is used. At first, we compare our CS-based TR approach using RF^{CS}_{avg}, called OURS in the following, to the FR-based approach of Jøsang from Definition 10, called JØSANG in the following. Given Alice and three sources, the fusion results are illustrated in Fig. 7. In the scenario, the sources S_0 and S_1 predict sunny weather, represented by the gray opinions $\omega_X^{S_0}$ and $\omega_X^{S_1}$, respectively, but Alice's trust in both sources, represented by the blue trust opinions $\omega^A_{S_0}$ and $\omega^A_{S_1}$, is reduced. Source S_2 predicts rainy weather, represented by the blue opinion $\omega_X^{S_2}$, and Alice has no doubts about this source (blue trust opinion $\omega^A_{S_2}$). As a result, Alice's trust-fused opinion about the current weather state is rather indecisive, represented by the blue opinion $\omega_X^{\diamond([A;\mathbb{S}])}$. Using JØSANG reduces the trust in S_0, represented by the orange trust opinion shift, as it is the only one with greater than average conflict with the FR; respectively, Alice's TR fused opinion tends more to rainy weather (orange opinion $\breve{\omega}_X^{\diamond([A;\mathbb{S}])}$). In contrast, using OURS implicitly favors the majority of source opinions towards sunny weather; thus, TR reduces the trust in S_2, represented by the green trust opinion shift, which is the only source predicting rain. Consequently, Alice's opinion about the weather changes towards sunny weather (green opinion $\tilde{\omega}_X^{\diamond([A;\mathbb{S}])}$).

In the same scenario, the behavior of the gTR can be demonstrated. It allows the use of both previously shown TR approaches simultaneously. The recommended weight limits are omitted for demonstration and illustration purposes, and the weights for both approaches are swept over the interval $[0, 5]$. While this leads to impractically high RF, it helps to show the behavior based on the relation of both weights. The result is shown in Fig. 8a. As required, by setting the values of α_{OURS} and $\alpha_{\text{JØSANG}}$, the result alters accordingly. JØSANG converges at the rightmost point and, thus, increasing its weight further has no effect. Additionally,



(a) gTR using OURS and JØSANG.



(b) gTR using OURS and MS-DIFF.

Fig. 8. TR results of the gTR approach are shown. The blue opinions represent the TF results without TR. Green/Orange opinions show the TF results using gTR. Along the orange direction $\alpha_{\text{JØSANG}}$ and $\alpha_{\text{MS-DIFF}}$ is increased, and, along the green direction α_{OURS}.

TABLE I

OPINIONS USED IN THE TIME-DEPENDENT EXPERIMENT.

	Alice			Sources		Const Pred.		Change Pred.	
	$\omega^{A_k}_{A_{k-1}}$	$\omega^{A}_{S^i_k}$	$\omega^{A}_{P_k}$	$\omega^{S^i_k}_{\text{rainy}}$	$\omega^{S^i_k}_{\text{sunny}}$	$\omega^{P_k}_{\text{rainy}}$	$\omega^{P_k}_{\text{sunny}}$	$\omega^{P_k}_{\text{rainy}}$	$\omega^{P_k}_{\text{sunny}}$
b_X	0.90	0.50	0.50	0.80	0.00	0.99	0.00	0.00	0.99
d_X	0.05	0.00	0.00	0.00	0.80	0.00	0.99	0.99	0.00
u_X	0.05	0.50	0.50	0.20	0.20	0.01	0.01	0.01	0.01
$a_X(x)$	1.00	1.00	1.00	0.50	0.50	0.50	0.50	0.50	0.50
$P_X(x)$	0.95	1.00	1.00	0.90	0.10	0.99	0.00	0.00	0.99

in Fig. 8b, results are illustrated using OURS and the multi-source uncertainty differential approach, called MS-UD. Here, the weight $\alpha_{\text{MS-UD}}$ is swept over $[0, 20]$. The results demonstrate that multiple TR approaches can be combined. Moreover, the use of gTR can be task-specifically adjusted. Thus, depending on the task at hand, the TR behavior can be adjusted to any weighted combination of multiple TR approaches.

B. Time-Dependent Subjective Networks

In this section, the time-dependent weather forecast example is evaluated. For simplicity reasons, the weather domain is $\mathbb{W} = \{\text{rainy}(belief), \text{sunny}(disbelief)\}$, the number of sources in each time step is constant, and Alice's trust in each source prior to TR is the same. Further, Alice's trust in her previous decision only depends on the time difference between steps. The sources observe a predefined opinion for rainy and sunny weather, respectively. A prediction either predicts a constant or a changing weather. All opinions are shown in Table I.

In the following, OURS is compared to JØSANG and to a fusion where no trust is considered. The prediction is applied in all three cases, and cumulative fusion is used. Our scenario starts with sunny weather. All sources report back to Alice accordingly. Next, at time step 5, one source is reporting rainy weather incorrectly. At time step 15, an incorrect change in weather is predicted. Then, at time step 30, the weather switches to sunny, which is correctly predicted. Finally, at time step 40, the weather switches back again but without being predicted. Results are shown in Fig. 9.

Generally, the small error of all approaches at time step 30 proves that a correct prediction allows a correct decision with fast changes in the observed state. However, whenever information is erroneous, differences in the approaches become visible. In all three cases of misinformation, OURS yields the lowest error compared to the ground truth (GT) information. This means that incorrect inputs have less influence while, at the same time, the decision adapts to changes more quickly compared to the other methods. While JØSANG reduces the influence of errors in some cases (time step 15, and 40), it increases the influence in others (time step 5). Here, the consideration of majority tendencies in OURS improves the overall decision and, thus, OURS is better suited for the task than JØSANG. This demonstrates the choice of the TR approach is of importance.

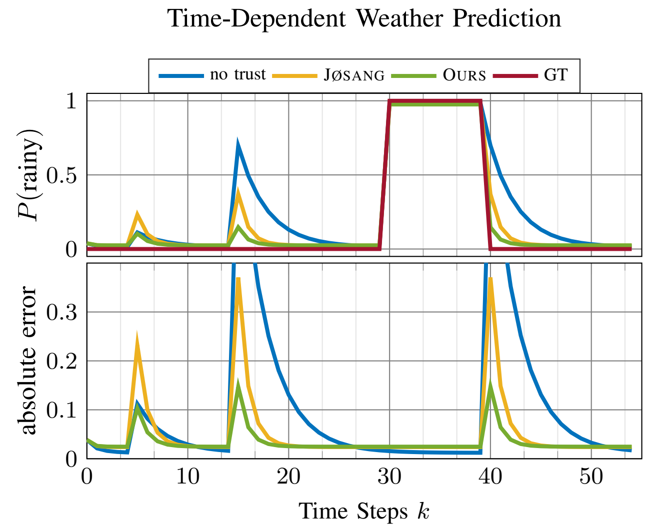


Fig. 9. The projected probability progression and the error with respect to the GT over time are shown for different settings. In red, the GT is shown; at time step 30, the weather switches from sunny to rainy, and at time step 40 back to sunny.

VI. CONCLUSION

Summarizing this work, we proposed a novel trust revision approach, demonstrated its differences and advantages from existing methods, and provided a generalized trust revision approach enabling the combined use of different trust revision operators. Additionally, we proposed rules under which a subjective network can be transformed into a recursive sub-subjective network. Compared to existing methods, they allow an efficient modeling and evaluation of time-dependent decision processes. In combination with the proposed trust revision approaches, this work extends the use of subjective networks and allows for efficient conflict handling in time-dependent decision processes.

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